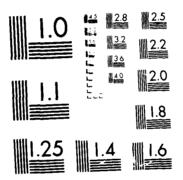
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DIRECT-SOLUTION TECHNIQUES FOR VISCOUS FLOWS AND THEIR CONTROL

K.N. GHIA
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U. GHIA



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#### DIRECT-SOLUTION TECHNIQUES FOR VISCOUS FLOWS AND THEIR CONTROL

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This research was supported by the Air Force Office of Scientific Research, Bolling Air Force Base, under AFOSR Grant No. 87-0074, with Dr. James D. Wilson as Technical Monitor.

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#### 19. ABSTRACT (Continued)

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ABSTRACT

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#### 1. RESEARCH OBJECTIVES

The objectives that were set for this grant consisted of the following:

(i) To examine and achieve active control of two-dimensional (2-D) unsteady separated flows through direct numerical simulation using the unsteady Navier-Stokes (NS) equations,

and

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(ii) To study three-dimensional (3-D) unsteady, separated vortex-dominated flows by developing a direct implicit technique; this should also permit analysis of flow instabilities using model problems.

The research pursued in both of these areas since October 1986 and the progress achieved is described briefly in the next section.

#### 2. RESEARCH PROGRESS

#### 2.1 Control of Separated Flows

Active control/management of separated flows is studied by considering the flow in a backstep channel with an oscillating flap located near the downstream end of the primary separation bubble on the lower wall as shown in Fig. 1. Time-dependent boundary-oriented clustered conformal coordinates were generated, during the first half of this reporting period, using the Schwarz-Christoffel conformal mapping, followed by suitable contraction mappings to resolve the critical regions in this flow. A brief description of the method for developing the transformation and determining the coordinates for arbitrary orientations of the oscillating flap is given in Appendix. During the second half of this reporting period, effort has been focussed on determination of the flow in the temporally deforming domain of this flow.

### 2.1.1 Governing Equations

The problem is represented mathematically by the vorticity-stream function formulation of the time-dependent Navier-Stokes equations for 2-2 incompressible flow. For a time-accurate solution of these equations for unsteady flow, the stream function  $\nu$  is obtained by a direct-solution method based on block-Gaussian elimination (BGE), for every time-step advance of the vorticity field  $\omega$ . With temporally deforming grids, even the linear differential equation for  $\nu$  takes on time-varying coefficients. Therefore, additional computational time is required not only for the evaluation of these coefficients but also for the solution of the corresponding partial differential equation by direct inversion. By contrast, for efficiency of the BGE procedure, the differential operator for  $\psi$  must remain time-invariant. Conformal mapping allows this to be achieved even in the presence of temporally deforming grids, as briefly described next.

In Cartesian coordinates, the stream-function equation has the form

$$\psi_{xx} + \psi_{yy} = -\omega \tag{1}$$

where the source function is the vorticity  $\omega$  which is a function of x, y and t, i.e.,

$$\omega = \omega(x,y,t) . \tag{2}$$

In conformal coordinates  $(\xi,\eta)$ , this equation can be represented as

$$\psi_{\xi\xi} + \psi_{\eta\eta} = -h^2\omega \tag{3}$$

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where

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$$h = h_1 = h_2 \tag{4}$$

is the scale factor of the conformal mapping

$$\xi + i\eta = f(x+iy) . (5)$$

In terms of complex variables, the conformal mapping and its scale factor

$$z = f(\zeta)$$
,  $h = \left| \frac{df}{d\zeta} \right| = \left| \frac{dz}{d\zeta} \right|$  (6a,b)

$$\zeta = \xi + i\eta \text{ and } z = x + iy$$
 (7a,b)

In terms of complex variable become  $z=f(\zeta) \ , \ k$  where  $\zeta=\xi+i\eta \ \ and$  For a temporally definition system, the scale factor varying. However, by viring equation (3) that results be absorbed in the source equation  $\psi_{\xi\xi}+\psi_{\eta\eta}=G(\xi,\eta)$ For a temporally deforming boundary-aligned conformal coordinate system, the scale factor h varies with time, in addition to being spatially varying. However, by virtue of the special form of the stream function equation (3) that results with a conformal mapping, this time-varying h can be absorbed in the source function of this equation leading simply to the

$$\psi_{\xi\xi} + \psi_{\eta\eta} = G(\xi, \eta, t) \tag{8a}$$

where

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$$G(\xi,\eta,t) \stackrel{\Delta}{=} - n^2 \omega . \tag{8b}$$

This leaves the stream-function equation with time-invariant coefficients. It is important to note that any clustering transformations, introduced in Eq. (8a) for the purpose of resolving the length scales of the problem, must be time-independent in order for the final form of the equation to possess time-independent coefficients. The clustering transformations used in the present work are of the form  $\xi = \xi(S)$ ,  $\eta = \eta(N)$  where (S,N) are the final computational coordinates.

The transport equation for the vorticity  $\omega$  for 2-D incompressible flow is written in vector form as

$$\omega_{t} + \nabla \cdot \nabla \omega = \frac{1}{Re} \nabla^{2} \omega .$$

Here,  $\overline{V}$  is the velocity vector with components u and v along the conformalcoordinate directions  $\xi$  and  $\eta$ , respectively, related to the stream function

$$u = \frac{1}{h} \frac{\partial \psi}{\partial \eta} \quad ; \qquad v = -\frac{1}{h} \frac{\partial \psi}{\partial \xi} \quad . \tag{10}$$

The time-dependent conformal mapping can be expressed as

$$\xi = \xi(x,y,t)$$

$$\eta = \eta(x,y,t)$$

where  $\tau$  is the tranformed time coordinate. Hence, the time derivative term

$$\omega_{t} = \omega_{\tau} + \xi_{t} \omega_{\xi} + \eta_{t} \omega_{\eta} \tag{11}$$

$$\omega_{t} = \omega_{\tau} + \frac{1}{h^{2}} \left[ (x_{\eta} y_{\tau} - y_{\eta} x_{\tau}) \omega_{\xi} + (x_{\xi} y_{\tau} - y_{\xi} x_{\tau}) \omega_{\eta} \right]. \tag{12}$$

 $\omega_t + \vec{V} \cdot \nabla \omega = \frac{1}{Re} \ \nabla^2 \omega \ .$  Here,  $\vec{V}$  is the velocity vector with coordinate directions  $\xi$  and  $\eta$ , results as follows:  $u = \frac{1}{h} \frac{\partial \psi}{\partial \eta} \ : \quad v = -\frac{1}{h} \frac{\partial \psi}{\partial \xi} \ .$  The time-dependent conformal mapping  $\xi = \xi(x,y,t)$   $\eta = \eta(x,y,t)$   $\eta = \eta(x,y,t)$   $\tau = t$  where  $\tau$  is the transformed time coordinate  $\xi$  in Eq. (9) transforms as  $\omega_t = \omega_\tau + \frac{\xi}{h^2} \int_{\xi} (x_\eta y_\tau - y_\eta x_\tau) \omega_\xi$  The spatial derivatives of x, y and  $\xi$  in Eq. (10) the grid speed test of the spatial derivatives of  $\xi$  in Eq. (11) and  $\xi$  in Eq. (11) and  $\xi$  in Eq. (12) and  $\xi$  in Eq. (13) and  $\xi$  in Eq. (14) and  $\xi$  in Eq. (14) and  $\xi$  in Eq. (15) and  $\xi$  in Eq. (15) and  $\xi$  in Eq. (16) and  $\xi$  in Eq. (17) and  $\xi$  in Eq. (17) and  $\xi$  in Eq. (18) and  $\xi$  in Eq. (19) and  $\xi$ The spatial derivatives of x, y are easily evaluated using the complex quantity dz/dz. The grid speed terms  $\boldsymbol{x}_{_{\boldsymbol{\tau}}},\;\boldsymbol{y}_{_{\boldsymbol{\tau}}}$  are determined using the mapping at two successive levels of time.

The wall boundary conditions are obtained from the condition of zero slip along the non-porous walls of the backstep channel. These are then expressed in terms of the stream function  $\psi$ , as follows:

Along 
$$n = n_{min}$$
 and  $n = n_{max}$ , 
$$\psi_e = -hv \; ; \qquad \psi_n = hu \; . \tag{13a,b}$$

For points on a stationary non-porous wall, these lead to the conditions that, at these points,

$$\psi \Big|_{\eta_S} \approx 0$$
 and  $\psi_{\eta} \Big|_{\eta_S} = 0$  (14a,b)

where  $\eta_S$  denotes stationary-boundary points. Equation (14a) implies that the boundary is a streamline. For points on a moving boundary, the right-hand sides of either Eq. (13a) or Eq. (13b) or both may be non-zero, even when the boundary is non-porous. This is easily confirmed by examining the streamline pattern around a cylinder moving at a constant velocity in an otherwise stagnant fluid (Fig. 2). If, at points A and B on the cylinder,  $\psi = \psi_1$  at A and  $\psi = \psi_2$  at B, then  $\psi$  is not constant on the cylinder surface. Hence, for the present problem,  $\psi$  along the moving flap is no longer zero even though it is zero elsewhere on the lower boundary of the channel. The distribution of  $\psi$  along the moving flap is obtained by using Eq. (13) together with information about the flap motion, i.e.,

Along the flap:

$$u_F = 0$$
 and  $v_F = r\dot{\theta}$  (15a,b)

where r is the distance along the flap measured from its base (Fig. 3), and  $\dot{\theta}$  is the instantaneous rate of its rotation. The specified motion of the flap will provide  $\dot{\theta}$ , whereas r is obtained from  $\xi$  and h for the conformal mapping. Use of Eqs. (15a,b) in Eqs. (13a,b) leads to

$$\psi_2 - \psi_1 = -\int_{\xi_1} h v_F d\xi \quad \text{and} \quad \psi_n |_{\eta_F} = 0$$
 (16a,b)

As regards concerns about accuracy of the integration indicated in Eq. (16a), it is important to note that  $\psi$  = 0 at points adjoining both sides of

the base of the flap. This provides a check on the integration accuracy and is used to formulate the discretized integration.

the base of the flap. This provides a check on is used to formulate the discretized integration.

Another concern that needs to be dealt with points in the computational plane are uniformly all time. With time-dependent clustering transf distribution in the doubly-infinite straight-cha all time. The corresponding distributions along plane at two successive time levels may not, in physical motion of the flap; see Fig. 4. For a treated without difficulty, using simple interpor resulting from possible binding, this requires of All of the above analysis has been incorpor and is being made operational. Some preliminary within the next few weeks, unless any additional and need to be resolved. Current plans are for to be presented at the National Fluid Dynamics C 1983, in Cincinnati, Ohio.

2.1.3 Flow Control for Axisymmetric Configuration For 2-D rectangular configurations, the structure processes constant coefficients. This is not so configurations where some of the coefficients in Stokes stream function contain the radius r in a given computational point, the value of r change shape alters. This implies that the Schwarz-Chridirectly as dz/d; needs to be integrated at ever this implies that the elimination phase of the repeated at every time step. The time-consuming Another concern that needs to be dealt with is the following. The grid points in the computational plane are uniformly distributed and fixed for all time. With time-dependent clustering transformations, the grid-point distribution in the doubly-infinite straight-channel plane is also fixed for all time. The corresponding distributions along the flap in the physical plane at two successive time levels may not, in general, be related by the physical motion of the flap; see Fig. 4. For a straight flap, this can be treated without difficulty, using simple interpolation. For curved flap resulting from possible binding, this requires careful consideration.

All of the above analysis has been incorporated into the flow program and is being made operational. Some preliminary results are anticipated within the next few weeks, unless any additional problems are encountered and need to be resolved. Current plans are for a paper based on this work to be presented at the National Fluid Dynamics Congress scheduled for July,

#### 2.1.3 Flow Control for Axisymmetric Configurations

For 2-D rectangular configurations, the stream function equation possesses constant coefficients. This is not so for axisymmetric configurations where some of the coefficients in the equation governing the Stokes stream function contain the radius r in an essential manner. For a given computational point, the value of r changes with time as the boundary shape alters. This implies that the Schwarz-Christoffel mapping obtained directly as dz/dζ needs to be integrated at every time step. Furthermore, this implies that the elimination phase of the BGE method needs to be repeated at every time step. The time-consuming nature of this phase has a large adverse effect on the efficiency of the overall procedure. The possible use of the MG-SI (Multi-Grid Strongly Implicit) procedure developed earlier by U. Ghia et al. [1983, 1987] is being explored for the stream function equation for the axisymmetric configuration of a pipe-orifice with bending of the orifice plate. The preliminary findings from this effort are summarized in Table I, and indicate that the MG-SI scheme is a viable procedure for solution of elliptic equations with time-dependent coefficients.

#### 2.2 Unsteady Free Stream - A Control Mechanism

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Aerodynamic forces can be significantly affected by the unsteady nature of the separation and reattachment phenomena on the suction surface of an aerodynamic body at high angle of attack. Present understanding of the underlying mechanisms is quite limited. A study of the flow past an elliptic cylinder at angle of attack was initiated around July 1987, with the objective of studying flow control by subjecting the elliptic cylinder to a time-dependent free stream.

As a first phase of this work, the flow past an elliptic cylinder is being studied, using the two-dimensional unsteady incompressible Navier-Stokes analysis of K. Ghia, Osswald and U. Ghia [1985]. Clustered conformal transformations are developed to obtain appropriate surface-oriented coordinates for an elliptic cylinder. The grid-clustering transformations implemented attempt to resolve the multiple disparate length scales of the unsteady separated flow; a typical grid consists of (229x45) mesh points.

The results of the present unsteady analysis are compared, qualitatively, with the experimental data of Izumi and Kuwahara [1983] in Figs. 5 and 6. For the case of an elliptic cylinder, with 50% thickness, angle of attack  $\alpha$  = 20° and Reynolds number Re = 369. Before discussing

these figures, it is appropriate to point out the conditions under which the two sets of data are obtained, so the qualitative comparison carried out is in proper perspective. In the experiments, an electrolytic precipation method was used to obtain flow visualization results; Taneda [1977] has shown that such an approach leads to visualization of 'integrated streaklines' which, in turn, show the 'vorticity' layers. Further, although the time scales are non-dimensionalized in the same way, the cylinder was accelerated over a finite amount of time in the experiment, whereas in the present analysis, the cylinder was impulsively started from rest. Finally, the orientation of the two sets of results shown is also different; the experimental photographs show a horizontal free stream and the cylinder at an angle of attack, but the present computational results are plotted with the axis of the cylinder being horizontal and the oncoming flow is at the appropriate angle of attack.

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In Fig. 5, the instantaneous flow field results for "vorticity" as stated above, are shown for characteristic time T=0.86. In the photograph as well as the plot, a counterclockwise rotating vortex is beginning to shed off of the lower surface near the trailing edge, while a clockwise rotating vortex sheet is reaching across the upper surface and curls down towards the trailing edge. In Fig. 6, the results are depicted at T=1.69. Both sets of results show that the counterclockwise rotating vortex, that was initially seen being shed at T=0.86, has moved downstream away from the cylinder. Figure 6 also shows the vortical structure positioned just over the upper portion of the trailing edge and the vortex sheet from the leading edge curling up and over this vortical structure in both the photograph as well as the plot of present results.

Preliminary results are obtained for flow past an elliptic cylinder with 25% thickness at an angle of attack  $\alpha$  = 15° and Re = 1000. The flow solutions monitored showed that the cylinder, which is impulsively started from rest, goes through a highly unsteady phase until T = 17, after which the asymptotic state in the form of a limit cycle is reached. Further detailed examination of the limit cycle solution showed alternate shedding of vortices from the suction and pressure surfaces. The instantaneous stream function and vorticity are plotted in Fig. 7 (a-d) for the above configuration at T = 28.47 corresponding to the maximum-lift instant and T = 29.106 for the minimum-lift instant in the same cycle. The corresponding instantaneous velocity vectors for the two cases are depicted in Fig. 7 (e-f).

### 2.3 Three-Dimensional Incompressible Separated Flow

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This study has continued using the model problem of the shear-driven flow within a 3-D rectangular cavity. Preliminary results using the direct inversion methodology for the unsteady Naver-Stokes equations were obtained during the first half of this report period culminating in the paper presented by Osswald, K. Ghia and U. Ghia [1987] at the 8th AIAA CFD Conference, Honolulu, Hawaii, appearing in the conference proceedings. Results obtained on the Department's Perkin Elmer 3250 using the serial research version of the code were presented for a (17x17x17) grid. This proved the formulation of the unsteady Navier-Stokes problem as well as the efficiency of the direct, inversion numerical procedures.

Since then the code has been fully vectorized for the CYBER 205 computer, resulting in efficient direct numerical simulation capabilities for nightly unsteady 3-D viscous flows. Memory management considerations are currently being investigated using a two pronged approach.

Direct, inversion methodologies trade memory utilization (a relatively inexpensive resource) for CPU efficiency (still a scarse resource). For fine 3-D grids (33x33x33) to (65x65x65) the coefficient matrices employed to gain the efficiency of the direct inversion procedures exceed the in-core memory conveniently obtainable on the CYBER 205 at NASA/Langley. A two-pronged approach is being pursued to circumvent this difficulty. the strong numerical stability of the direct solution procedures, demonstrated in Osswald et al. [1987], permits the storage of these coefficient matrices in half-precision (32 bit) words rather than the fullprecision (64 bit) words currently employed. This effectively doubles the in-core storage conveniently obtainable on the CYBER 205 system for holding the coefficient matrices without further recoding. In addition, use of the concurrent I/O capability of the CYBER 205 system is being investigated with the concept of implementing program controlled concurrent paging through this data base simultaneously with the solution calculations. This could greatly reduce the in-core storage requirement of the direct solution techniques placing the burden instead upon the concurrent I/O processor. Finally, access to the CRAY-2 facility at NASA/Ames is being pursued where significantly more memory is available to the single user.

Results of this memory management exercise as well as fine grid flow results for the driven cavity and 3-D backstep channel geometries has been submitted for presentation at the ICNMFD '88 conference at Williamsburg, Virginia. The flow inside a backstep channel with finite spanwise extent will be examined next. For this geometry, careful laser-Doppler velocimetry results are available for the entire range of flow, i.e., for laminar, transition and turbulent flows.

#### 3. RESEARCH FORECAST

As described in the preceding section, both areas of research pursued have progressed well to meet the objectives of this study. During the second year of the grant period, i.e. October 15, 1987 - October 14, 1988, effort will continue as follows:

- a. The active-flow reattachment-control study will be completed by carrying out a parametric study of the backstep channel flow with control flap to achieve optimum control. This effort will be coordinated with that of Nagib at the Illinois Institute of Technology. Various scenarios of flap geometry, location and motion will be examined in order to determine the optimum configuration of this flow-control mechanism.

  Control of unsteady separated flow will also be examined via the elliptic cylinder configuration with an unsteady free stream.
- b. Concurrently, the 3-D incompressible unsteady flow analysis and solution procedure will be applied to the flow in a rectangular cross-section duct with a backward facing step and verified by comparing the predictions with the corresponding laser-Doppler velocimetry (LDV) data available from the University of Karlsruhe and Stanford University in order to obtain insight into the mechanism of shear-layer instability, transition and the strange-attractor theory of turbulence.

Effort will also be directed towards gaining better familiarity with effective usage of the IRIS workstation. This equipment is especially useful in the present unsteady flow research, particularly, the 3-D work.

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#### 7. SCIENTIFIC INTERACTIONS - SEMINARS AND PAPER PRESENTATIONS

#### Invited Lectures:

- Ghia, K.N., presented at the Second Nobeyama Workshop on Fluid Dynamics and Supercomputers, Tokyo, Japan, 1987.
- Ghia, U., presented at the <u>Second Nobeyama Workshop on Fluid Dynamics and Supercomputers</u>, Tokyo, Japan, 1987.
- Ghia, K.N. and Ghia, U., presented at International Symposium on Computational Fluid Dynamics, Sydney, Australia, August 1987.
- Ghia, K.N., presented Two Lectures in CFD Lecture Series at Indian Institute of Technology, Bombay, India, August 1987.
- Ghia, U., presented Two Lectures in <u>CFD Lecture Series</u> at Indian Institute of Technology, Bombay, India, August 1987.
- Ghia, K.N. and Ghia, U. presented at <u>DARPA-URI Symposium on Periodic and</u>
  Aperiodic Phenomenon Behind Circular Cylinders, Newport, RI, July 1987.
- Ghia, K.N. presented at Department of Mechanical Engineering and Applied Mechanics, University of Rhode Island, Providence, RI, July 1987.
- Ghia, K.N. presented at <u>University of California at Davis</u>, Davis, California, June 1987.

#### Paper Presentations

- Ghia, K.N., Ghia, U. and Osswald, G.A., "Analysis of Low-Speed Multidimensional Unsteady Viscous Flows," presented at Workshop II on Unsteady Separated Flows, Colorado Springs, Colorado, July 1987.
- Ghia, U., Ramamurti, R. and Ghia, K.N., "Study of Viscous Flows with Upstream Interaction Using Interacting PNS Equations," presented at the R.T. Davis Symposium on Computational Mechanics, Cincinnati, Ohio, June 1987.
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- Zuo, L., Ghia, U. and Ghia, K.N., "Efficient Numerical Grid Generation Methods Based on Conformal Mapping," presented at AIAA 13th Annual Minisymposium on Aerospace Science and Technology, Dayton, Ohio, March 1987.
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TABLE I

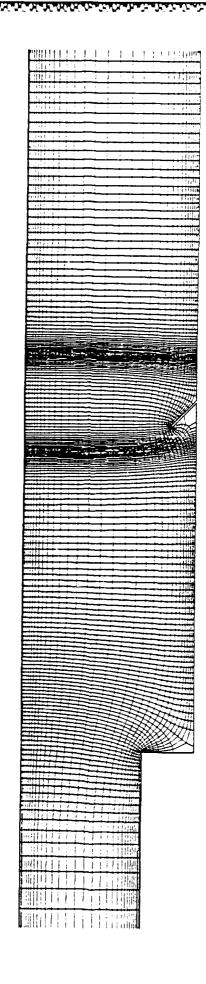
# Computational Effort Units for BGE and MG-SI Solution of Stream-Function Equation for Axisymmetric Flow

Method	Phase 1	Phase 2	Total
BGE	6	1	7
MG-SI	2	2	4

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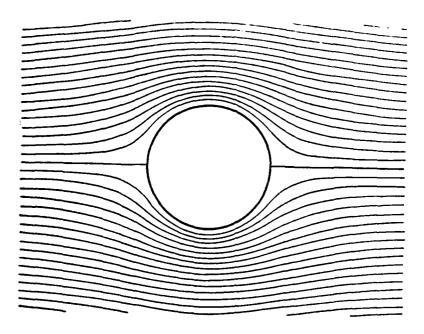
Note 1: The effort quoted is referred to the CPU time required for Phase 2 of the BGE method for the axisymmetric orifice-pipe problem using a (257x33) grid.

Note 2: The test was conducted at the initial stage of the flow problem. During later stages, the MG-SI-phase 2 should require a reduced CPU time, whereas the BGE CPU time always remains fixed.

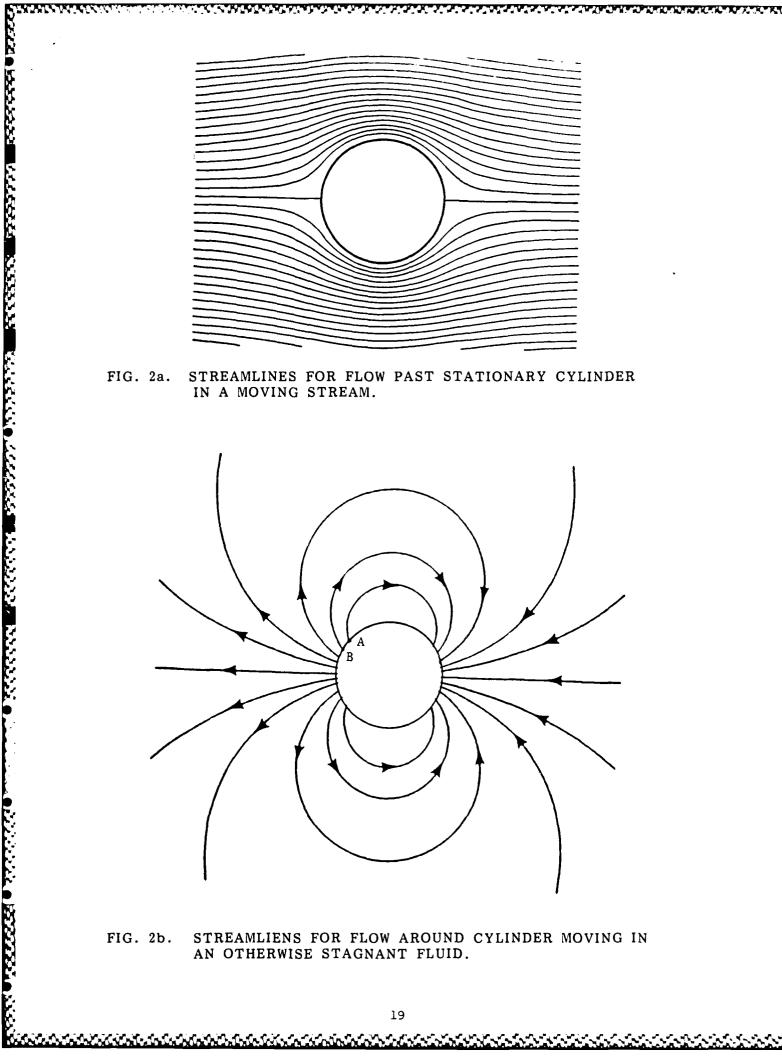


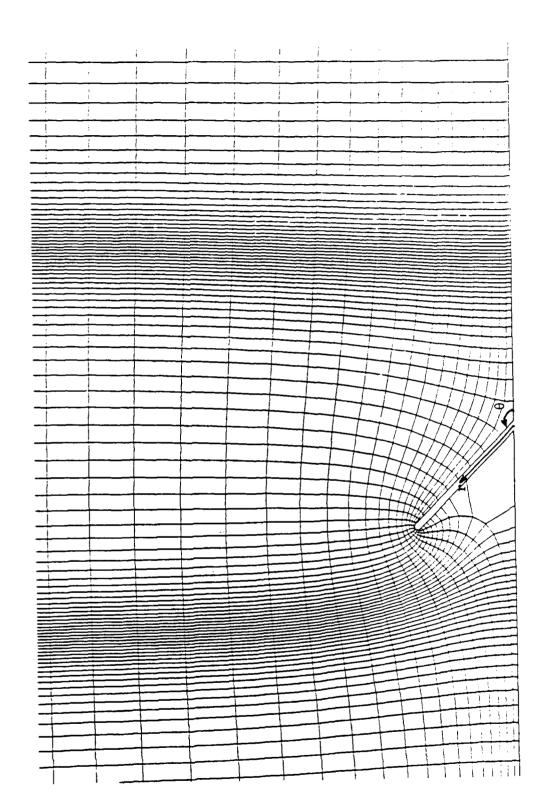
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CLUSTERED CONFORMAL BOUNTARY-ALIGNED GRID FOR BACKSTEP CHANNEL WITH OSCILLATING FLAP. FIG. 1.



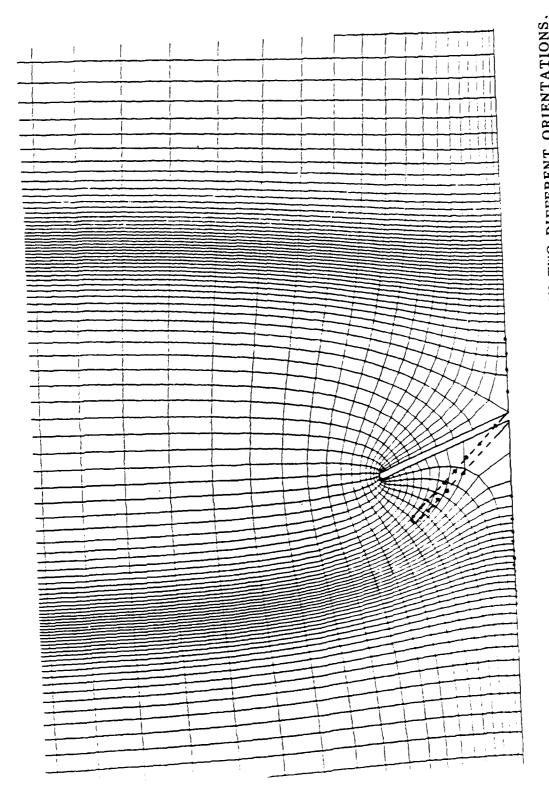
STREAMLINES FOR FLOW PAST STATIONARY CYLINDER FIG. 2a. IN A MOVING STREAM.





LOCAL POLAR COORDINATES FOR OSCILLATING FLAP. FIG. 3.

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LOCATIONS OF BOUNDARY POINTS ALONG FLAP IN TWO DIFFERENT ORIENTATIONS. FIG. 4.



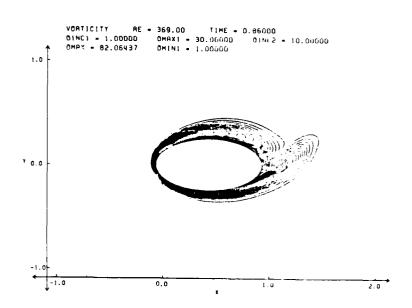


FIG. 5. COMPARISON OF "VORTICITY" CONTOURS AT Re = 369,  $\alpha$  = 20°, T = 0.86, (a) DATA OF IZUMI AND KUWAHARA, (b) RESULTS OF PRESENT ANALYSIS.



\$2550 | \$255500 | \$255550 | \$455500 | \$355500 | \$155500 | \$155500 | \$155500 | \$255550 | \$155500 | \$155500 | \$155500 |

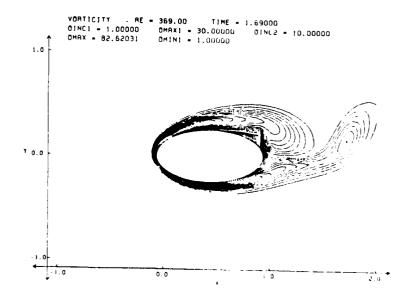
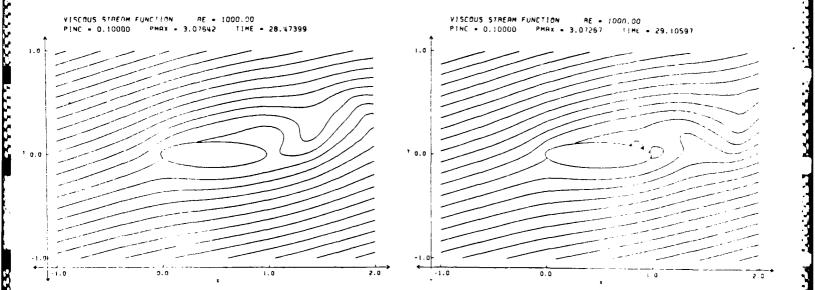


FIG. 6. COMPARISON OF "VORTICITY" CONTOURS AT Re = 369,  $\alpha$  =  $20^{\circ}$ , T = 1.69, (a) DATA OF IZUMI AND KUWAHARA, (b) RESULTS OF PRESENT ANALYSIS.



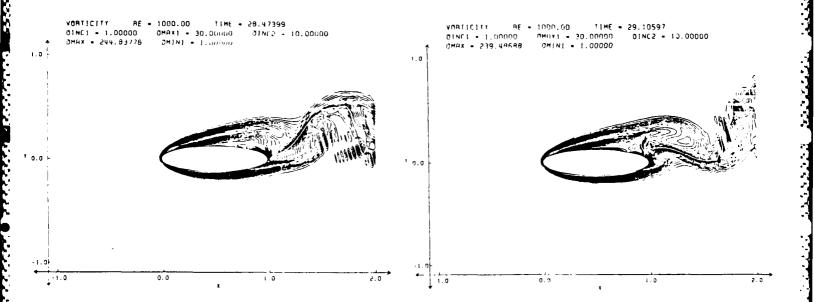
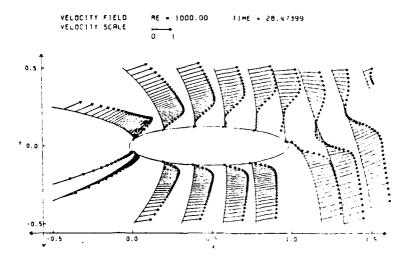


FIG. 7. FLOW FIELD RESULTS FOR Re = 1000,  $\alpha$  = 15°. (a,b) INSTANTANEOUS STREAM FUNCTION, (c,d) INSTANTANEOUS VORTICITY CONTOURS



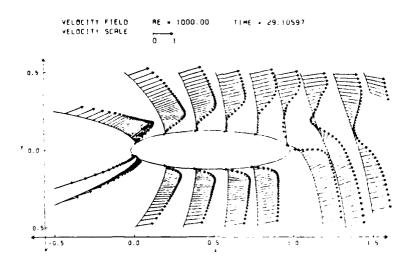


FIG. 7 (Cont'd.). FLOW FIELD RESULTS FOR Re = 1000,  $\alpha$  = 15°. (e,f) INSTANTANEOUS VELOCITY VECTORS.

#### APPENDIX

#### EFFICIENT GENERATION OF CONFORMAL COORDINATES FOR ARBITRARY GEOMETRIES

#### A.1 Outline of Conformal Mapping Employed

Following the work of Davis [1983], the Schwarz-Christoffel conformal transformation given by the equation

$$\frac{dz}{d\zeta} = M \prod_{i=1}^{n} (\zeta - a_i)^{\alpha_i/\pi}$$
(A.1)

is employed to map an n-sided polygon in the z-plane onto the upper half of the  $\zeta$ -plane as shown in Fig. A.1. In Eq. (A.1), the  $\alpha_i$ 's are known from the given geometry, while M and the  $a_i$ 's are unknown.

Since it is more appropriate to transform an arbitrary channel to a straight channel rather than to an upper-half plane, a further transformation is introduced which conformally maps the upper-half  $\zeta$ -plane to a straight channel in the T-plane. Clearly, this latter transformation is simply a special case of the Schwarz-Christoffel transformation [Eq. (A.1)] and is given as

$$\frac{d\zeta}{dT} = \pi \zeta \quad , \tag{A.2a}$$

so that

$$T = \frac{1}{\pi} \ln(\zeta) \quad \text{or} \quad \zeta = \exp(\pi T) \quad . \tag{A.2b}$$

Hence, the overall transformation is obtained as

$$\frac{dz}{dT} = \pi \zeta \frac{dz}{d\zeta} \bigg|_{\zeta = \exp(\pi T)}$$
 (A.3)

with  $dz/d\zeta$  given by Eq. (A.1). Further details associated with the actual determination of this transformation are presently being documented in a paper (U. Ghia, Zuo and K. Ghia [1988]) to be submitted for publication in a technical journal. This work also constitute the basis of a Master's degree

with dz/dz given by Eq. (A.1). Further details associated determination of this transformation are presently being do paper (U. Ghia, Zuo and K. Ghia [1983]) to be submitted for technical journal. This work also constitute the basis of thesis (Zuo [1987]).

Figure A.2 shows typical grids obtained, using this and backstep channel with an oscillating flap. Two representates the flap are shown. The grid clustering occurring on the list unnecessary for these positions of the flap. However, is because the grid-clustering transformations are time-invariable clustering shown will be needed when the flap rotates to be in the second quadrant with reference to the axis of the flap is a small price to pay for the important advantage that the equation retains time-invariant coefficients and can continue fficiently by the BGE direct method.

A.2 Enhancement of Efficiency for Evaluation of Transformations the transformation means to be accomplianted for Figure A.2 shows typical grids obtained, using this analysis, for the backstep channel with an oscillating flap. Two representative positions of the flap are shown. The grid clustering occurring on the left of the flap is unnecessary for these positions of the flap. However, it is retained because the grid-clustering transformations are time-invariant and the clustering shown will be needed when the flap rotates to occupy a position in the second quadrant with reference to the axis of the flap motion. This is a small price to pay for the important advantage that the stream-function equation retains time-invariant coefficients and can continue to be solved

#### A.2 Enhancement of Efficiency for Evaluation of Transformation

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Since the transformation needs to be re-evaluated for every incremental change in orientation of the flap, considerable effort was focused on enhancing the efficiency of determination of the transformation. The results of this effort are summarized in Table A.1. Prior to commenting on these results, it is important to note the following. The Schwarz-Christoffel transformation directly provides the complex derivative dz/dz or dz/dT and, hence, the scale factor h of this conformal mapping. The flow equations contain only h and its derivatives. Therefore, integration of  $dz/d\zeta$  itself is needed only when it is desired to view the grid. This may

not be necessary at each and every time step of the flow simulation; only h needs to be evaluated at every computational time level.

For channels, Eq. (A.3), together with Eq. (A.11), possesses a simple asymptotic form as  $Re(\zeta) \to \pm \infty$ . These equations approach this asymptotic form in an exponential manner, so that the simplified form constitutes an accurate representation of the actual transformation beyond relatively small distances away from the last deviation from the straight-channel configuration. As shown in Table A.1, implementation of this asymptotic form resulted in 66-80 percent saving of CPU time for the computation of  $h^2$ , while the CPU time saved in the grid calculation was almost 80-95 percent when the grid is obtained by integrating dz/dT in the T-plane, i.e., in the straight-channel plane. If the integration is done in the computational plane of the clustered coordinates (S,N), the CPU time saved is not nearly as large, but the grid clustering obtained in the physical plane is better.

In Table A.1, item B refers to the actual coding employed for evaluating the multi-exponential terms appearing in Eq. (A.1). After examining the manner in which such terms are actually evaluated in the computer, the coding was modified to reflect this manner of evaluation. This resulted in an additional 50 percent saving in the CPU time required to evaluate  $h^2$ . Hence, the implementation of both items A and B resulted in a total CPU time saving of almost 90 percent. It is believed that this level of efficiency is acceptable for proceeding on with the flow-control study.

A paper based on this efficient grid-generation study is presently under preparation by U. Ghia et al. [1988]. This study should also be very useful in various related areas such as those occurring in problems involving fluid-structure interactions and free surfaces, metal-forming problems as well as in bio-fluid mechanics involving flow through passages

with flexible walls. All of these problems encounter temporally deforming boundaries and may often require the consideration of more than one discipline of study. For example, in fluid-structure interaction problems, the fluid dynamics analysis provides the time-varying force distribution, while the structural analysis provides the boundary deformations (vibrations) occurring under the action of these unsteady forces. The high efficiency of the grid-generation method, together with the present formulation of the stream-function equation with constant coefficients even in the presence of temporally deforming geometry, paves the way for effectively examining the oscillating-flap flow-control mechanism.

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TABLE A-1
ENHANCEMENT OF EFFICIENCY

	PERCENTAGE OF COMPUTATIONAL TIME SAVED		
	h <sup>2</sup> CALCULATION	GRID CALCULATION	
BY IMPLEMENTING			
ITEM 'A'	66 - 80	80 - 95* (66 - 80) <sup>+</sup>	
ITEM 'B'	50	0	
ITEMS 'A'+'B'	83 - 90	80 - 95* (66 - 80) <sup>+</sup>	

'A': ASYMPTOTIC FORM FOR CHANNEL MAPPING.

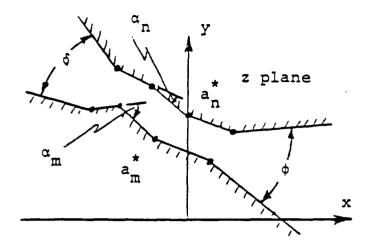
'B': TIME-SAVING TECHNIQUE FOR EVALUATING

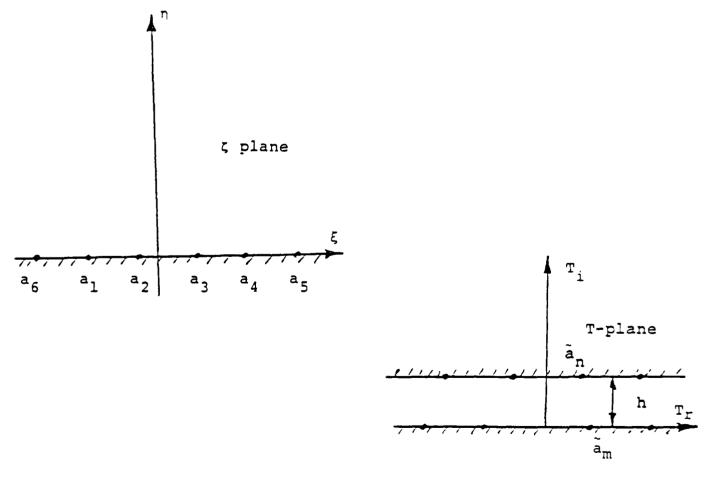
MULTI-EXPONENTIAL TERMS.

NOTE: CURVED BOUNDARIES ARE ALSO TREATED WITH

COMPARABLE EFFICIENCY.

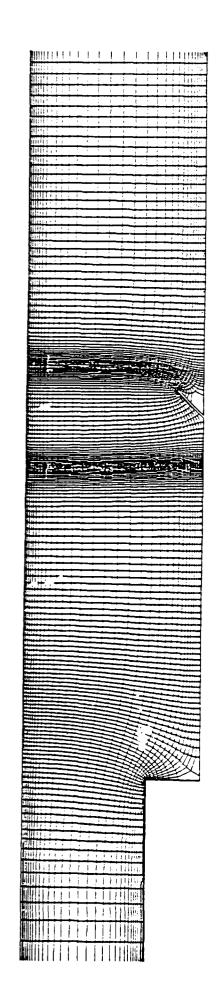
- \* INTEGRATION DONE IN CHANNEL PLANE (T).
- + INTEGRATION DONE IN COMPUTATIONAL PLANE (S,N).





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FIG. A-1 CONFORMAL MAPPING OF ARBITRARY CHANNEL TO STRAIGHT CHANNEL.



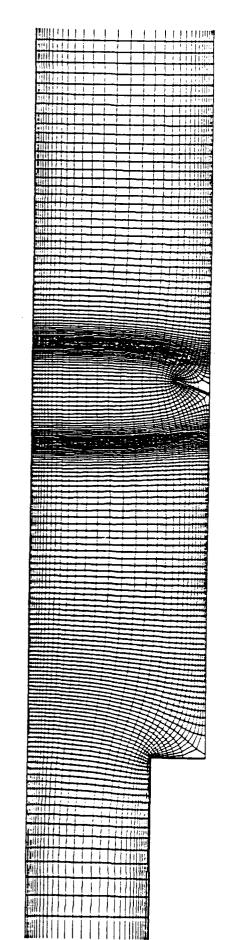


FIG. A-2 CLUSTERED CONFORMAL BOUNDARY-ALIGNED GRID FOR BACKSTEP CHANNEL WITH OSCILLATING FLAP IN TWO DIFFERENT POSITIONS.

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